Dynamic Simulation of Multiple Effect Evaporators in Paper Industry Using MATLAB

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Abstract— **The present study attempts to develop a dynamic model for the MEE to study the transient behavior of the system. Each effect in the process is represented by a number of variables which are related by the energy and material balance equations for the feed, product and liquor flow. In the present study dynamic equations are being written for the MEE system for a Paper Industry. In this study a generalized model is given which could be applied to any number of effects in the MEE system with simple modifications it could also be applied for Forward Feed, Backward Feed as well as for the Mixed Feed. For such situation basic equations for an effect will be same but the equations for the parameters like density, boiling point elevation and specific heat etc. should be changed and then the model can be used for the other type of evaporator also.**

Keywords - **Multiple effect evaporator (MEE); backward feed; Runge-Kutta method; boiling point elevation (BPE), multiple effect distillation (MED).**

I. INTRODUCTION

Mathematical Modeling is an indispensable tool to analyze, correlate, simulate, optimize, and finally control any chemical process system. Pulp and paper industry is very capital intensive industry and consists of many subsystems. Huge amount of raw material, chemical, energy and water are consumed in the process of paper manufacture with requirement of large labor force. Systematic mathematical modeling of each subsystem is therefore an imperative necessity to solve the above issues one after another. One such energy intensive subsystem called evaporator is used to concentrate black liquor from pulp mill. Multiple Effect Evaporators (MEE) dealing with the concentration of weak black liquor from pulp mill of paper industry consume a large amount of thermal energy in form of steam (25-40%).

A wide variety of mathematical models for multiple effect evaporators can be found in the scientific literature. Normally the main difference among these mathematical models is the heuristic knowledge which is incorporated in their development.

El-Nashar [1] developed a simulation model for predicting the transient behavior of ME stack type distillation plants. Transient heat balance equations were written for each plant component in terms of the unknown temperatures of each effect. The equations were solved simultaneously to yield the time-dependent effect of temperature as well as performance ratio and distillate production. The results of the simulation program were compared with actual plant operating data taken during plant start-up, and agreement was found to be reasonable.

Lambert, Joyo and Koko [6] developed a system of non-linear equations governing the MEE system and presented a calculation procedure for reducing this system to a linear form and solved iteratively by the Gaussian elimination technique. Boiling point rise and nonlinear enthalpy relationships in temperature and composition were included. The results of linear and nonlinear techniques were compared.

Hanbury [11] presented a steady-state solution to the performance equations of an MED plant. The simulation was based on a linear decrease in boiling heat transfer coefficient.

Miranda and Simpson [10] describe a phenomenological, stationary and dynamic model of a multiple effect evaporator for simulation and control purposes. The model includes empirical knowledge about thermo physical properties that must be characterized into a thermodynamic equilibrium. The properties selected evolved from an economical optimization because of their influence on the temperature and concentration variations parameters. The developed model consists of differential and algebraic equations that are validated using a parameter sensitivities method that uses data collected in the industrial plant. The simulation results show a qualitatively acceptable behavior.

Tonelli, Romagnoli, and Porras [12] presented a computed package for the simulation of the open-loop dynamic response of MEE for the concentration of liquid foods. It is based on a non-linear mathematical model. An illustrative case study for a triple effect evaporator for apple juice concentrators was presented. The response of the unit to large disturbances in steam pressure and feed flow rate based on the solution of the mathematical model was in excellent agreement with the experimentally determined response.

Narmine and Marwan [4] developed a dynamic model for the MEE process to study the transient behavior of the system. This model allowed the study of system start up, shut down, load changes and troubleshooting in which plant performance changed significantly. Each effect in the process is represented by a number of variables which are related by the energy material balance equation for the feed, product and brine flow. These equations were solved simultaneously to predict the system time dependent parameters under various transients. The effect of feed flow, feed temperature, live steam flow changes on plant performance such as temperature, brine salinity, product flow rate and brine level was investigated to test the validity of the model.

In the present investigation the dynamic model of MEE system of a paper industry is developed by using energy and material balance equations to study the transient behavior of the system. Further the parametric equations were solved for steady state data and the system of simultaneous ordinary differential equations for sextuple backward feed evaporator to study the transient behavior of the system.

II. MATHEMATICAL MODELING

The Mathematical modeling is carried out for sextuple backward feed evaporator system. In backward feed evaporator system the steam input is given in the first effect and the feed input is in the last effect. The material and energy flow for ith effect is given in the Fig-1. Model equations are presented for ith effect using material and energy balance equations and equations stating the physical properties of the black liquor. It is assumed that the vapor generated by the process of concentration of black liquor is saturated. It is also assumed that the energy and mass accumulation in the vapor lump is neglected as it is very small as compared to the enthalpy of the steam.

Material balance for liquor in the i^{th} effect:

$$\frac{d}{dt}Ml(i)(t) = Wl(i+1) - Wl(i) - Wv(i) \qquad (1)$$

Energy balance for liquor in the i^{th} effect:

$$\frac{d}{dt}\left(Ml(i)(t) * hl(i)(t)\right) = Wl(i+1)hl(i+1) - Wl(i)hl(i) - Wv(i)hv(i) + Wv(i-1)hv(i-1) - Wv(i-1)hc(i-1) \qquad (2)$$

Material balance for solids in ith effect:

$$\frac{d}{dt}\left(Ml(i)(t) * X(i)(t)\right) = Wl(i+1)X(i+1) - Wl(i)X(i) \qquad (3)$$

Where Ml(i) can be written as:

$$Ml(i) = A\,L(i)Pl(i) \qquad (4)$$

The Pl(i) is the density of the liquor given by[3]

$$Pl(i) = (997 + 649X(i))[1008 - 0.237(Tl(i)/1000) - 1.94(Tl(i)/1000)^2] \qquad (5)$$

Where Tl(i) is in °C

The vapor and liquor in ith effect are in equilibrium and the relation for the liquor and vapor temperature is defined in terms of boiling point elevation (BPE) is as follows:

$$Tl(i) = Tv(i) + BPE(i) \qquad (6)$$

and the boiling point elevation (BPE) is given by [3]

$$BPE(i) = (6.173X(i) - 7.48X(i)^{1.5} + 32.747X(i)^2) * (1 + 0.6(Tv(i) - 3.7316)/100) \qquad (7)$$

Where Tv(i) is in °K.

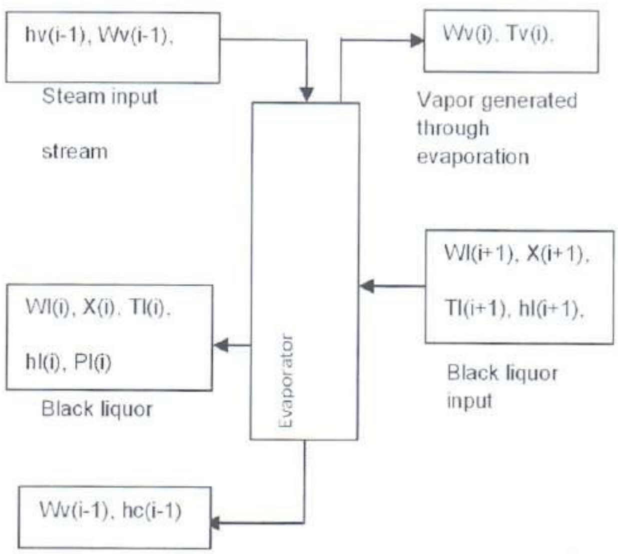

Fig-1 Block Diagram with terms used for the i^{th} effect

Specific heat of water at constant pressure, Cp given by [3]

$$Cp(i) = 4.216(1 - X(i)) + (1.675 + 3.31Tl(i)/1000) * X(i) + (4.87 - 20Tl(i)/1000) * (1 - X(i)) * X(i)^3 \quad (8)$$

Where Tl(i) is in °K.

Differentiating equation (6) with respect to time we get

$$\frac{d}{dt}Tl(i)(t) = \left(\frac{d}{dt}Tv(i)(t)\right)\left\{1 + \left(\frac{\partial}{\partial Tv}BPE(i)\right)\right\} + \left(\frac{\partial}{\partial X}BPE(i)\right)\left(\frac{\partial}{\partial t}X(i)(t)\right) \quad (9)$$

Differentiating equation (4) with respect to time and using the value of the value of $\frac{d}{dt}Tl(i)(t)$ from equation (9) and by rearranging the terms we get the value of $\frac{d}{dt}Ml(i)(t)$ and then comparing the resultant differential equation with equation (1) we get

$$C1 = C2\left(\frac{d}{dt}L(i)(t)\right) + C3\left(\frac{d}{dt}Tv(i)(t)\right) + C4\left(\frac{d}{dt}X(i)(t)\right) \quad (10)$$

Where :

$$C1 = Wl(i+1) - Wl(i) - Wv(i)$$

$$C2 = APl(i)$$

$$C3 = AL(i)\left(\frac{\partial}{\partial Tl}Pl(i)\right)\left(1 + \left(\frac{\partial}{\partial Tv}BPE(i)\right)\right)$$

$$C4 = AL(i)\left(\left(\frac{\partial}{\partial Tl}Pl(i)\right)\left(\frac{\partial}{\partial X}BPE(i)\right) + \left(\frac{\partial}{\partial X}Pl(i)\right)\right)$$

Differentiating Ml (i) (t)*hl (i) (t) with respect to time and using the value of $\frac{d}{dt}Tl(i)(t)$ from equation (9) and the value of $\frac{d}{dt}Ml(i)(t)$ from the equation which is obtained by the differentiation of equation (4) and then comparing the resultant differential equation with equation (2) we get

$$C5 = C6\left(\frac{d}{dt}L(i)(t)\right) + C7\left(\frac{d}{dt}Tv(i)(t)\right) + C8\left(\frac{d}{dt}X(i)(t)\right) \quad (11)$$

Where:

$$C5 = Wl(i+1)hl(i+1) - Wl(i)hl(i) - Wv(i)hv(i) + Wv(i-1)hv(i-1) - Wv(i-1)hc(i-1)$$

$$C6 = APl(i)hl(i)$$

$$C7 = AL(i)\left(1 + \left(\frac{\partial}{\partial Tv}BPE(i)\right)\right)\left(Pl(i)\left(\frac{\partial}{\partial Tl}hl(i)\right) + hl(i)\left(\frac{\partial}{\partial Tl}Pl(i)\right)\right)$$

$$C8 = AL(i)\left[\begin{array}{l} Pl(i)\left(\frac{\partial}{\partial Tl}hl(i)\right)\left(\frac{\partial}{\partial X}BPE(i)\right) + \\ Pl(i)\left(\frac{\partial}{\partial X}hl(i)\right) + hl(i)\left(\frac{\partial}{\partial Tl}Pl(i)\right) \\ \left(\frac{\partial}{\partial X}BPE(i)\right) + hl(i)\left(\frac{\partial}{\partial X}Pl(i)\right) \end{array}\right]$$

Differentiating Ml(i)(t)*X(i)(t) with respect to time and using the value of $\frac{d}{dt}Ml(i)(t)$ from the equation which is obtained by the differentiation of equation (4) and rearranging the equation and representing it in the form of coefficients

$$\frac{d}{dt}X(i)(t) = C9 + C10\left(\frac{d}{dt}L(i)(t)\right) + C11\left(\frac{d}{dt}Tv(i)(t)\right) \quad (12)$$

Where:

$$C9 = \frac{Wl(i+1)X(i+1) - Wl(i)X(i)}{AL(i)\left[Pl(i) + X(i)\left\{\left(\frac{\partial}{\partial Tl}Pl(i)\right)\left(\frac{\partial}{\partial X}BPE(i)\right) + \left(\frac{\partial}{\partial X}Pl(i)\right)\right\}\right]}$$

$$C10 = \frac{Pl(i)X(i)}{L(i)\left[Pl(i) + X(i)\left\{\left(\frac{\partial}{\partial Tl}Pl(i)\right)\left(\frac{\partial}{\partial X}BPE(i)\right) + \left(\frac{\partial}{\partial X}Pl(i)\right)\right\}\right]}$$

$$C11 = \frac{X(i)\left(\frac{\partial}{\partial Tl}Pl(i)\right)\left\{1 + \left(\frac{\partial}{\partial Tv}BPE(i)\right)\right\}}{Pl(i) + Xl(i)\left[\left(\frac{\partial}{\partial Tl}Pl(i)\right)\left(\frac{\partial}{\partial X}BPE(i)\right) + \left(\frac{\partial}{\partial X}Pl(i)\right)\right]}$$

III. MODEL VALIDATION

System of simultaneous ordinary differential equations given by equations (10), (11) and (12) is solved by Runge-Kutta method of 4th order. Runge-Kutta's methods do not require the calculations of higher order derivatives and give greater accuracy. Runge-Kutta's formulae has advantage of requiring only the function values at some selected points. Error of 4th order Runge- Kutta's Method is of the order of h5. A computer code is developed in MATLAB for the solution.

For the validation of the model, steady state data is calculated and compared with the data of the paper mill. The steady state results are in good agreement with the data of the paper mill.

IV. SIMULATION

A. Effect of varying feed flow rate:

To study the transient behavior a step change of a 10% disturbance in the liquor flow rate to the last effect was applied to the model and the response of the system toward this change is shown through the graphs of the various system variables from Fig-2 to Fig-7. Thus the feed flow rate is decreased by 10% and the response of it is seen in following variables.

- First Effect Temperature
- First Effect Liquor Level
- Product Flow
- Last Effect Temperature
- Last Effect Liquor Level
- Product Concentration

B. Effect of varying steam flow rate:

Similarly a step change of 10% in the steam flow rate to the first effect was applied and the response of the system toward this change is shown through the graphs of the various system variables from the Fig-8 to Fig-13. The feed flow rate is decreased by 10% and the response of it is seen in following variables.

- First Effect Temperature
- First Effect Liquor Level
- Product Flow
- Last Effect Temperature
- Last Effect Liquor Level
- Product Concentration

These graphs are drawn with respect to time and by the graphical study it could easily be estimated that how the variables approach to the new conditions from the previous one.

V. RESULTS AND DISCUSSION

A. Effect of varying feed flow rate

The MEE system is backward feed; hence the input liquor enters the evaporator system from the last effect. The variation in the feed flow effects the last effect variables more than it affects the previous effects variables. The steady state is reached more quickly in last effect from other effects due to the same reason. Hence we can say that the response of disturbance in feed flow is greater in last effect than in other effects as inferred by the results shown in the graphs.

B. Effect of varying steam flow rate

In MEE with backward feed; the steam enters the evaporator system from the first effect. The variation in the steam flow effects the first effect variables more than it affects the subsequent effects variables. The steady state is reached later in first effect from other effects due to the same reason. Hence we can say that the response of disturbance in steam flow is greater in first effect than in other effects as inferred by the results shown in the graphs.

VI. CONCLUSION

A dynamic mathematical model was developed for a sextuple backward feed evaporator for concentrating the black liquor by using material, energy balance equations and parametric correlations. The model is successfully validated using the data obtained from the mill. The transient behavior of liquor temperature, liquor level, and product flow & product concentration was studied by disturbing the liquor flow rate and steam flow rate by 10%.

The transient study shows that the liquor temperature and product flow rate first increase then decrease to reach a steady state with a decreasing feed flow rate, while they directly decrease before reaching steady state with a decreasing steam flow rate. The product concentration increases and reaches steady state with decreasing feed flow rate, while it decreases with decreasing steam flow rate. The variation in the feed flow affects the last effect variables more than it affects the previous effects variables and the steady state is reached quickly in last effect than other effects. The variation in the steam flow affects the first effect variables more than it affects the subsequent effects variables. The steady state is reached quickly in first effect from other effects.

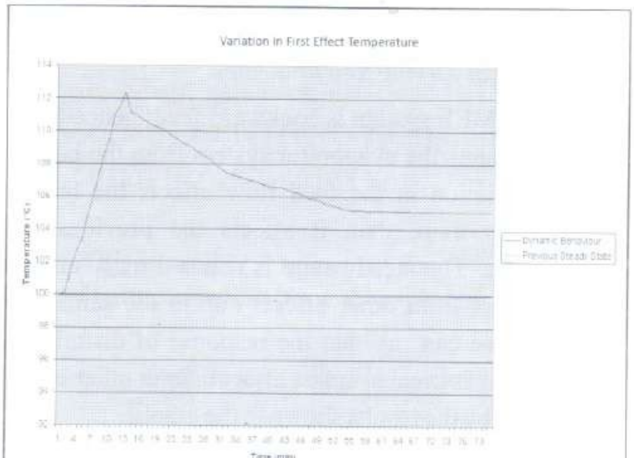

Fig-2 Variation in first effect temperature

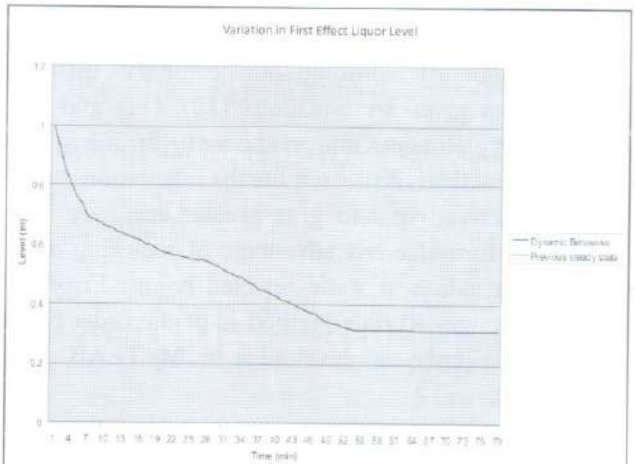

Fig-3 Variation in first effect temperature

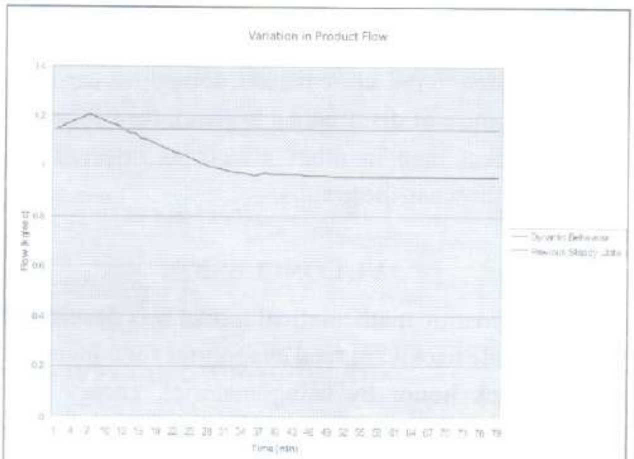

Fig-4 Variation in product flow

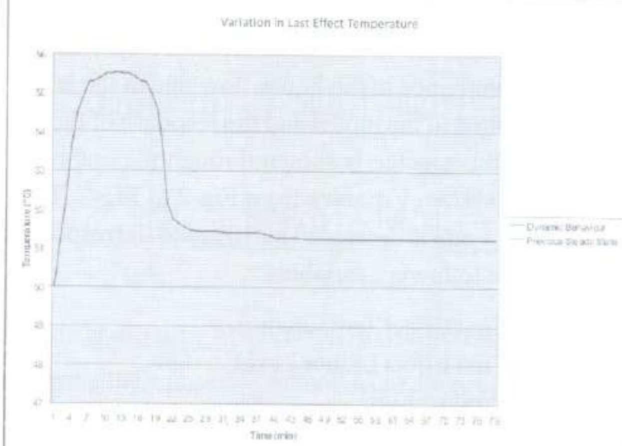

Fig-5 Variation in last effect temperature

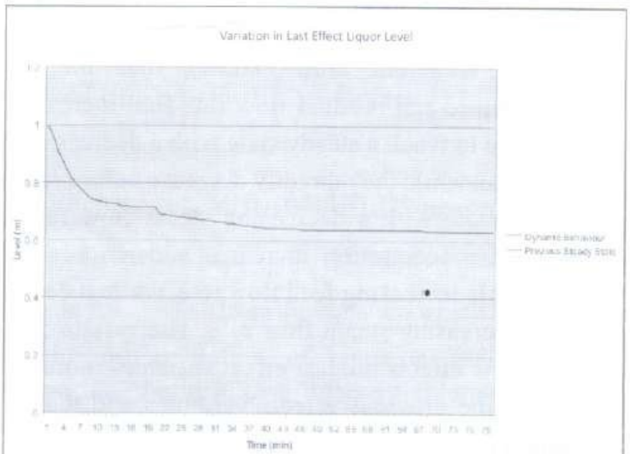

Fig-6 Variation in last effect liquor level

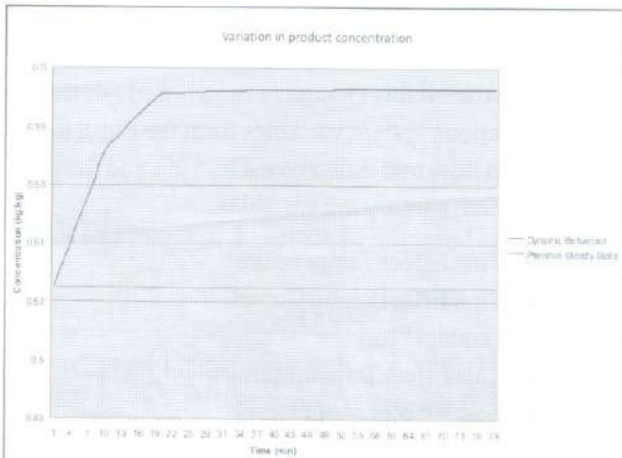

Fig-7 Variation in product concentration

A. Effect of varying feed flow rate Inference from the Graphs (Fig. 2 to 7)

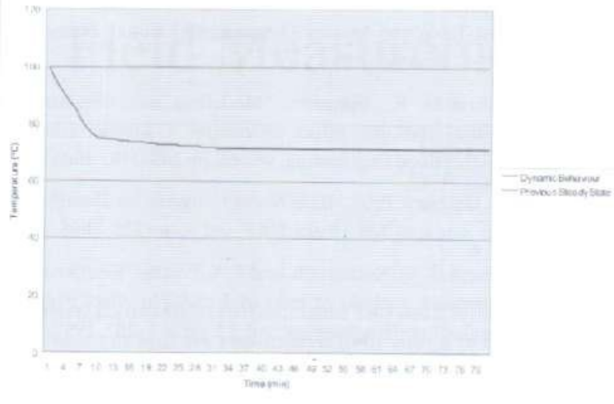

Fig-8 Variation in first effect temperature

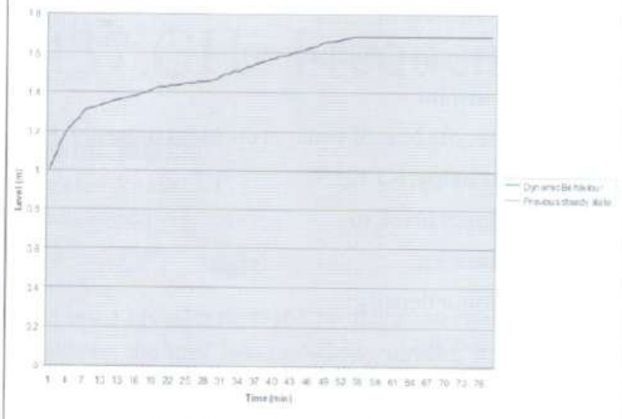

Fig-9 Variation in first effect liquor level

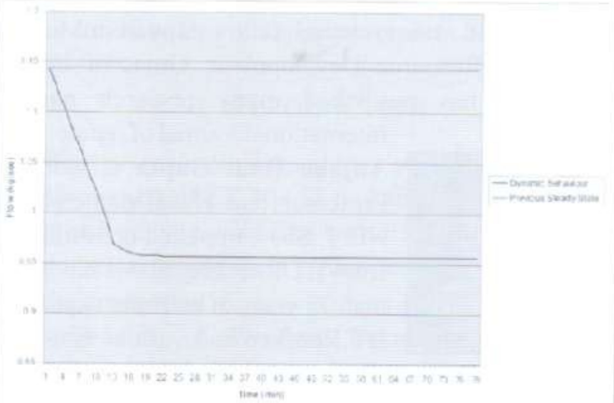

Fig-10 Variation in product flow

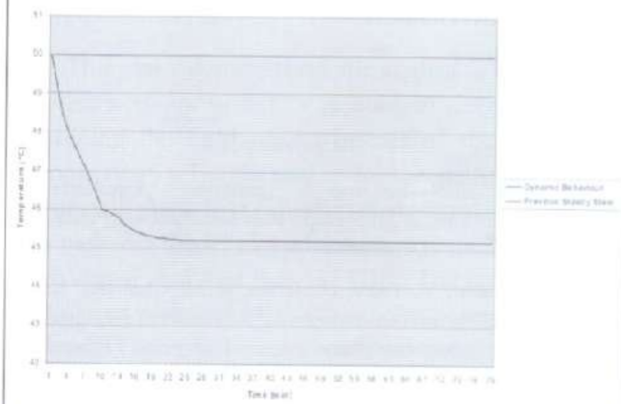

Fig-11 Variation in last effect temperature

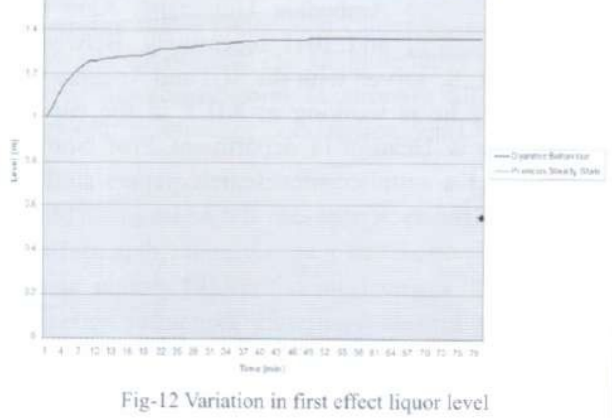

Fig-12 Variation in first effect liquor level

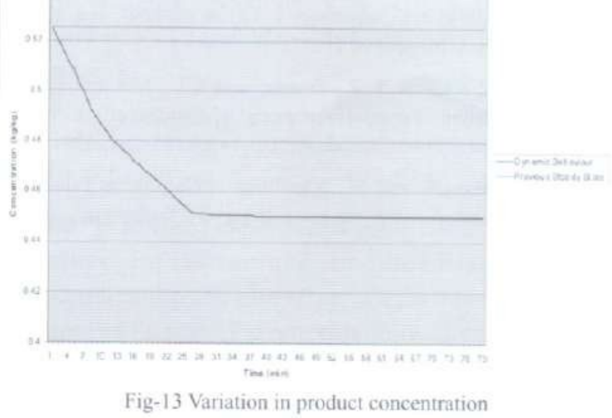

Fig-13 Variation in product concentration

B. Effect of varying steam flow rate Inferences from the graphs (Fig. 8 to 13)

NOMENCLATURE

A - Shell area, m^2

BPE - Boiling point elevation, °C

C - Constant

Cp - Specific heat of water at constant pressure, KJ/Kg

h - Enthalpy, KJ/Kg °C

L - Liquor level, m

M - Mass, kg

Pl - Liquor density

t - Time, sec

T - Temperature, °C

W - Mass flow rate, Kg/s

X - Solid content, %

Symbols used with the above terms:

c - Condensate

l - Liquor

i - Effect number

v - Vapor

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