Propagation Characteristics with Fractional Power in Core-Cladding of Optical Waveguide Using Helical Signal

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***Abstract*- Optical fibers are structures that are typically designed to transmit energy along a specified trajectory with minimal attenuation and single distortion. Optical fibers with small losses appeared in 1970. This event has laid the foundation for modern optical communication industry. However Si waveguides have some nonlinearity which occurs when electromagnetic waves interact with core of the waveguide. To do away with this problem it was begun to use Optical fibers with helical winding known as complex optical waveguides. Also in conventional waveguide modes are mixed with adjacent mode if V number is greater than 2.405. The use of helical winding in optical fibers makes the analysis much accurate. As the number of propagating modes depends on the helix pitch angle, so helical winding at core-cladding interface can control the dispersion or propagation characteristics of the optical waveguide. The model dispersion characteristics of circular optical waveguide with helical winding at core-cladding interface are obtained for different pitch angle. This paper gives the idea to obtain dispersion characteristics, and comparison of dispersion characteristics at different pitch angles. We obtained the dispersion characteristics by using boundary condition and this condition have been utilized to get the model Eigen values equation. From these Eigen value equations dispersion curve are obtained and plotted for five particular values of the pitch angle of the winding. Also fractional power flow in core and cladding has been calculated and the result has been compared.**

***Keywords*-Optical fiber communication, helical winding, optical fiber dispersion, fractional core and cladding power and helix pitch angle.**

I. INTRODUCTION

An optical waveguide (conventional fiber) is a cylindrical dielectric waveguide (non-conducting waveguide) with a circular cross section and ideally has a cylindrical shape. It consists of a core made up of a dielectric material having high refractive index (n1) which is surrounded by a cladding made up of a dielectric material having lower refractive index (n2) [1]. Even though light will propagate along the fiber core without the layer of cladding material, the cladding does perform some necessary functions. The cladding is generally made of glass or plastic. Optical fifers with helical winding are known as complex optical waveguides. The use of helical winding in optical fibers makes the analysis much accurate [1]. As the number of propagating modes depends on the helix pitch angle [2], so helical winding at core-cladding interface can control the dispersion characteristics [3-7] of the optical waveguide. The winding angle of helix (ψ) can take any arbitrary value between 00 to 900. In case of sheath helix winding [1], cylindrical surface with high conductivity in the direction of winding which winds helically at constant pitch angle (ψ) around the core cladding boundary surface. A sheath helix [1] can be approximated by winding a very thin conducting wire around the cylindrical surface so that the spacing between the adjacent windings is very small and yet they are insulated from each other. We assume that the waveguide have real constant refractive index of core and cladding is n1 and n2 respectively (n1 > n2). In this type of optical wave guide which we get after winding, the pitch angle controls the model characteristics of optical waveguide.

II. THEORETICAL ANALYSIS

We can take a case of a fiber with circular cross-section wound with a sheath helix at the core-clad interface (Fig. 1). A sheath helix can be assumed by winding a very thin conducting wire around the cylindrical surface so that the spacing between the nearest two windings is very small and yet they are insulated from each another. In our structure, the helical windings are made at a constant helix pitch angle (ψ). We assume that $(n1-n2)/n1 \ll 1$.

III. BOUNDARY CONDITIONS

The tangential component of the electric field in the direction of winding should be zero, and tangential component of both the electric field and magnetic field in the direction perpendicular to the winding must be continuous. So we consider the following boundary conditions [8].

$$Ez1 \sin\psi + E\phi 1 \cos\psi = 0 \quad (1)$$

$$Ez2 \sin\psi + E\phi 2 \cos\psi = 0 \quad (2)$$

$$(Ez1 - Ez2) \cos\psi - (E\phi 1 - E\phi 2) \sin\psi = 0 \quad (3)$$

$$(Hz1 - Hz2) \sin\psi + (H\phi 1 - H\phi 2) \cos\psi = 0 \quad (4)$$

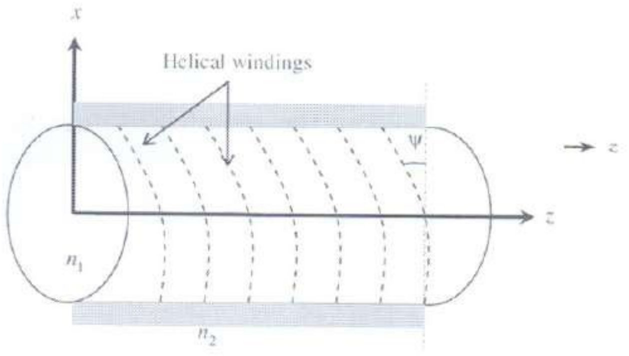

Fig. 1 Circular Optical Waveguide with conducting helical winding at core cladding interface

IV. OPTICAL WAVEGUIDE WITH HELICAL SIGNAL

The guided modes with this type of fiber can be analyzed in cylindrical coordinate system (r, φ, z). Where z is the direction of wave propagation i.e. along the axis of the optical fiber. The most important condition to have guided field is, $n2k < \beta < n1k$ and must be satisfied, where n1 and n2 are the refractive indices of the core and cladding region respectively and k is free space propagation constant ($k = 2\pi/\lambda$, $k2 = n2k$ and $k1 = n1k$). In core region we take the solution of linear combination of Bessel function of first kind {Jν (x)}, whereas for cladding region we take modified Bessel function of second kind {Kν (x)} [9]. We take ν = 1, for lower order guided mode index. The axial field components for core region can be can written as

$$E_{z1} = AJ_1(ua)\, F(\phi)\, e^{j(\omega t-\beta z)} \quad (5)$$

$$H_{z1} = BJ_1(ua)\, F(\phi)\, e^{j(\omega t-\beta z)} \quad (6)$$

The axial field components for clad region can be can written as

$$E_{z2} = CK_1(wa)\, F(\phi)\, e^{j(\omega t-\beta z)} \quad (7)$$

$$H_{z2} = DK_1(wa)\, F(\phi)\, e^{j(\omega t-\beta z)} \quad (8)$$

Also,

$$u^2 = k_1^2 - \beta^2 = \omega^2\mu\varepsilon_1 - \beta^2 \quad (9)$$

$$w^2 = \beta^2 - k_2^2 = \beta^2 - \omega^2\mu\varepsilon_2 \quad (10)$$

Where β is the axial component of propagation vector, ω is the wave frequency; μ is the permeability of the non-magnetic medium, ε_1 and ε_2 are permittivity of the core and cladding region respectively, and A, B, C and D are unknown constant and F (φ)is the function of coordinate φ. Now we are using Maxwell's equation to obtain transverse components of the electric field and magnetic field. So transverse components of the electric and magnetic field Eφ1 and Hφ1 for core region can be written as

$$E_{\phi 1} = -(j/u^2)[j(\beta/a)AJ_1(ua) - \mu\omega u BJ'_1(ua)]F(\phi)e^{j(\omega t-\beta z)} \quad (11)$$

$$H_{\phi 1} = -(j/u^2)[j(\beta/a)BJ_1(ua) + \omega\varepsilon 1 u AJ'_1(ua)]F(\phi)e^{j(\omega t-\beta z)} \quad (12)$$

And transverse components of the electric and magnetic field $E_{\phi 1}$ and $H_{\phi 1}$ for cladding region can be written as

$$E_{\phi 2} = -(j/w^2)[j(\beta/a)CK_1(wa) - \mu\omega w DK'_1(wa)]F(\phi)e^{j(\omega t-\beta z)} \quad (13)$$

$$H_{\phi 2} = -(j/w^2)[j(\beta/a)DK_1(wa) + \mu\omega w CK'_1(wa)]F(\phi)e^{j(\omega t-\beta z)} \quad (14)$$

Now eliminate the field components Eφ1, Hφ1, Eφ2, and Hφ2 from boundary conditions (1) to (4) and field component equations (11) to (14). We get four equations which involves four unknown constants A, B, C and D. Now we put coefficient of these unknown constants A, B, C and D into determinant to solve these four equations.

$$\begin{vmatrix} A1 & B1 & C1 & D1 \\ A2 & B2 & C2 & D2 \\ A3 & B3 & C3 & D3 \\ A4 & B4 & C4 & D4 \end{vmatrix} = 0$$

Now put $\Delta = 0$. This will produce non-trivial solution.

$$\Delta = 0 \tag{15}$$

Where A1 to A4 are coefficients of A, B1 to B4 are coefficients of B, C1 to C4 are coefficients of D and D1 to D4 are coefficients of D.

After simplifying the determinant, we get a simplified equation for lowest order modes.

$$u\frac{J_v(ua)}{J_v'(ua)}\left(\sin\psi + \frac{\beta v}{u^2 a}\cos\psi\right)^2 - \frac{k_1^2}{u}\frac{J_v'(ua)}{J_v(ua)}\cos^2\psi$$
$$-w\frac{K_v(wa)}{K_v'(wa)}\left(\sin\psi + \frac{\beta v}{w^2 a}\cos\psi\right)^2 + \frac{k_2^2}{w}\frac{K_v'(wa)}{K_v(wa)}\cos^2\psi = 0 \tag{16}$$

We use equation (16) to plot dispersion characteristics of an optical waveguide with helical winding. We can plot dispersion characteristics for different pitch angles (ψ). However the equation is valid for any value of pitch angle (ψ). We can also use equation (16) to find the value of β.

V. RESULT AND DISCUSSION

We plot the propagation or dispersion characteristics (V versus b) of the waveguide with helical winding. So we find the different value of β by using equation (16) for five different pitch angles (ψ) between 00 and 900. Here V is called normalized frequency [10] and determines how many modes a fiber can support. V is given by,

$$V^2 = (u^2 + w^2)a^2 = \left(\frac{2\pi a}{\lambda}\right)^2 (n_1^2 - n_2^2) = \left(\frac{2\pi a}{\lambda}\right)^2 NA^2 \tag{17}$$

Relation between β/k and V is given by normalized propagation constant (b), and is given by,

$$b = \left(\frac{aw}{V}\right)^2 = \left(\frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}\right) \tag{18}$$

VI. DISPERSION CHARACTERISTICS

Now we plot the propagation or dispersion characteristics for five different values of pitch angle as shown in Fig. 2, 3, 4, 5 and 6. For this we use n1 = 1.5, n2 = 1.46, and the $\lambda = 1.55$ µm.

1) *Dispersion Characteristics at Pitch Angle $\psi = 0^0$*: First we consider helical pitch angle $\psi = 0^0$. It means winding is perpendicular to the axis of the fiber, we can see obtained cut-off values for some modes as shown in dispersion curve (Fig. 2).

2 *Dispersion Characteristics at Pitch Angle $\psi = 30^0$*: In this case (in Fig. 3) we found that on the left of the lowest cut-off values, portions of curves appear which have no resemblance with standard dispersion curves, and have no cut-off values. This means that for very small value of V anomalous dispersion properties may occur in helically wound waveguides. Dispersion curve corresponding to Eq. 16 is shown in Fig. 3

3) *Dispersion Characteristics at Pitch Angle $\psi = 45^0$*: In this case (in Fig. 4) we found that on the left of the lowest cut-off values, portions of curves appear which have no resemblance with standard dispersion curves, and have no cut-off values. This means that for very small value of V anomalous dispersion properties may occur in helically wound waveguides. Dispersion curve corresponding to Eq. 16 is shown in Fig. 4

4) *Dispersion Characteristics at Pitch Angle $\psi = 60^0$*: In this case (in Fig. 5) we found that on the left of the lowest cut-off values, portions of curves appear which have no resemblance with standard dispersion curves, and have no cut-off values. This means that for very small value of V anomalous dispersion properties may occur in helically wound waveguides. Dispersion curve corresponding to Eq. 16 is shown in Fig. 5.

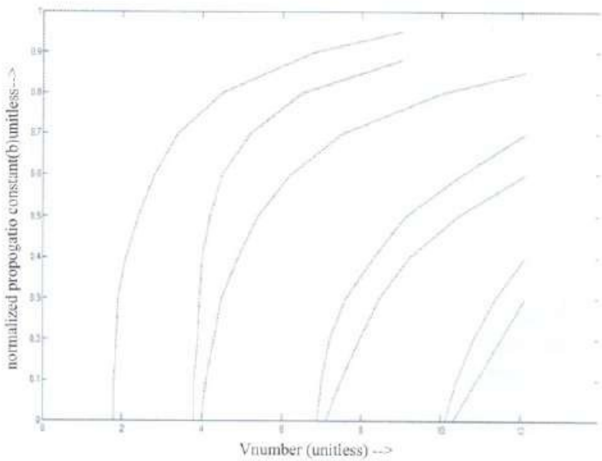

Fig. 2 Dispersion Curve for pitch angle $\psi = 0^0$

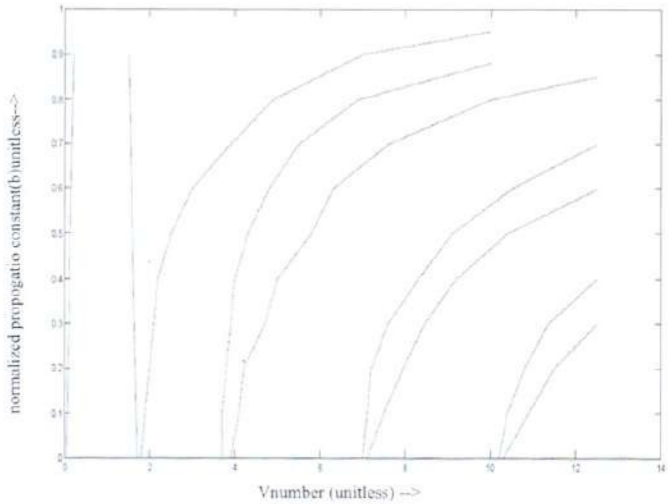

Fig. 3 Dispersion Curve for pitch angle $\psi = 30^0$

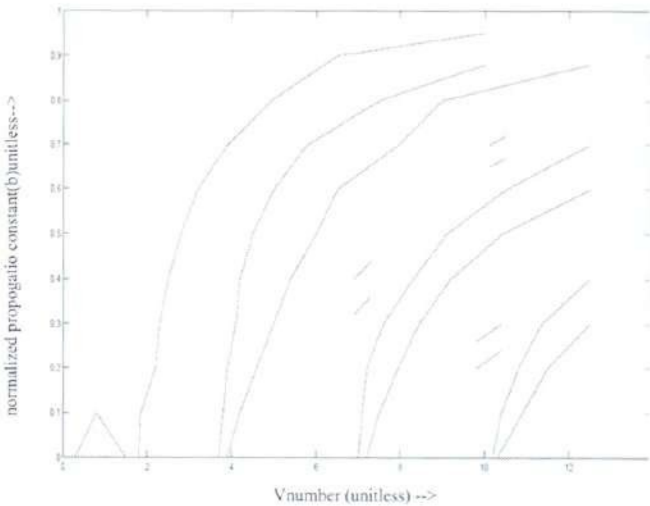

Fig. 5 Dispersion Curve for pitch angle $\psi = 60^0$

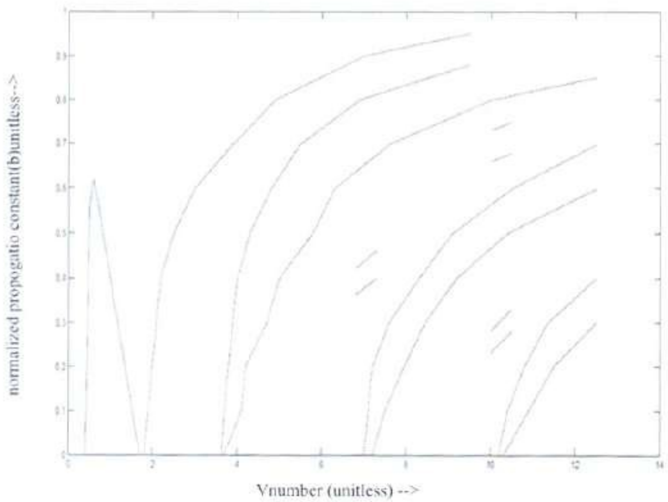

Fig. 4 Dispersion Curve for pitch angle $\psi = 45^0$

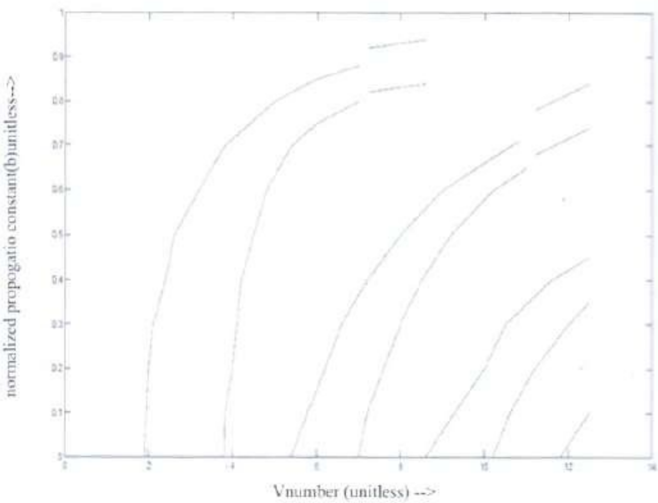

Fig. 6 Dispersion Curve for pitch angle $\psi = 90^0$

5) *Dispersion Characteristics at Pitch Angle $\psi = 90^0$*: It means winding is parallel to the axis of the fiber and have standard expected shape, but except for lower order modes they comes in pairs as shown in Fig. 6, that is cut-off values for two adjacent mode converge. We can also see obtained cut-off values for some modes as shown in dispersion curve Fig. 6. We observed that these two curves have different cut-off values. We observed that the cut-off value for helical pitch angle $\psi = 90^0$ is somewhat higher than that for helical pitch angle $\psi = 0^0$

VII. DEPENDENCE IN CUT-OFF VALUE V

In single mode fibers, V is less than or equal to 2.405. When V is 2.405, single mode fibers propagate the fundamental mode down the fiber core, while high-order modes are lost in the cladding. For low V values, most of the power is propagated in the cladding material. Power transmitted by the cladding is easily lost at fiber bends. The value of V should remain near the 2.405 level [10].

The cut-off value (V) is proportional to ψ and V number can also be used to express the number of modes M in a multimode fiber when V is large($V \geq 2.405$) [10]. For this case, an estimate of the total number of modes supported in a fiber is,

$$M \approx \frac{1}{2}\left(\frac{2\pi a}{\lambda}\right)^2 (n_1{}^2 - n_2{}^2) = \frac{V^2}{2}$$

Since the field of a guided mode extends partly into the cladding, as shown in Fig. 7, a final quantity of interest for a step-index fiber is the fractional power flow in the core and cladding for a given mode. As the V number increases no. of

TABLE I

Values of some lower-order modes (M) for different pitch angle (ψ)

Ψ	M	M	M	M	M	M	M
0^0	3.92	7.2	8.00	23.80	25.20	50.00	53.04
30^0	4.00	7.52	8.82	23.94	25.92	50.50	54.08
45^0	4.06	7.52	9.68	24.15	28.12	50.50	54.08
60^0	4.14	7.52	13.00	24.36	31.20	51.00	54.08
90^0	4.20	7.60	14.58	24.50	36.98	52.02	69.62

modes also increases as shown in table 1.

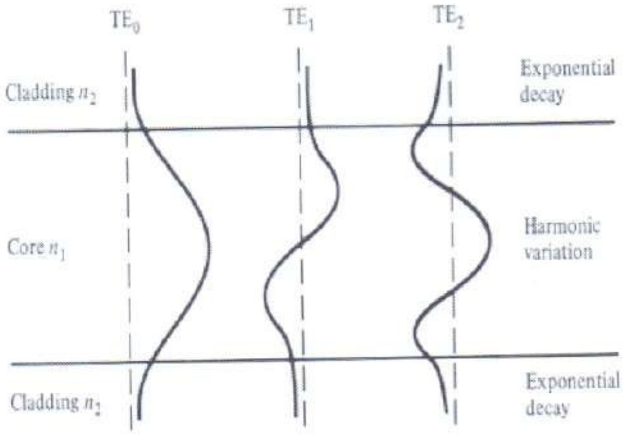

Fig. 7 Electric field distributions for several of lower-order guided modes

Far from cut-off-that is, for large values of V, the fraction of the average optical power residing in the cladding can be estimated by [10]

$$\frac{P_{clad}}{P} \approx \frac{4}{3\sqrt{M}}$$

Where; P is the total optical power in the fiber. Note that since M is proportional to V2, the power flow in the cladding decrease as V increases as shown in table 2 which is desirable and no doubt is increases in core (Pcore_P = 1 - Pcladd_P) as shown in table 3. However, this increases the number of modes in the fiber, which is not desirable for a high bandwidth [2].

TABLE 2

Fractional cladding power (Pcladd_P = P2) values for some lower-order modes

Ψ	P2	P2	P2	P2	P2	P2	P2
0^0	0.67	0.49	0.47	0.27	0.26	0.18	0.18
30^0	0.66	0.48	0.44	0.27	0.26	0.18	0.18
45^0	0.66	0.48	0.42	0.27	0.25	0.18	0.18
60^0	0.65	0.48	0.36	0.27	0.23	0.18	0.18
90^0	0.65	0.48	0.34	0.26	0.21	0.18	0.15

VIII. CONCLUSION

From the above results (Fig. 2, 3, 4, 5 and 6) we observe that, they all have standard expected shape, but except for lower order modes they comes in pairs, that is cut-off values for two adjacent mode converge. This means that one effect of conducting helical winding is to split the modes and remove a degeneracy which is hidden in conventional waveguide without windings.

We also observe that another effect of the conducting helical winding is to increase the cut-off values, thus increasing the number of modes. This effect is undesirable for the possible use of these waveguide for long distance communication.

We found that some curves have band gaps of discontinuities between some value of V. These represent the band gaps or forbidden bands of the structure. These are induced by the periodicity of the helical windings. Due to these band gaps wave propagate near the surface of core cladding interface. Hence there is decrease in power loss in cladding region. Thus helical pitch angle controls the modal properties of this type of optical waveguide.

TABLE 3

Fractional core power (Pcore_P = P1) values for some lower-order modes

Ψ	P1	P1	P1	P1	P1	P1	P1
0°	0.32	0.50	0.52	0.72	0.73	0.81	0.81
30°	0.33	0.51	0.55	0.72	0.73	0.81	0.81
45°	0.33	0.51	0.63	0.72	0.74	0.81	0.81
60°	0.34	0.51	0.63	0.72	0.76	0.81	0.81
90°	0.34	0.51	0.65	0.73	0.78	0.81	0.84

Also, addition of helical signals increase the acceptance angle, hence this increases the amount of light capacity of a waveguide and converts non uniform input signal into uniform output signal.

And last we found that the power flow in the cladding decrease as V increases as shown in table 2.

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