An Analysis of Utilization Factor of Fee-Counter in Colleges

*Nidhi Sharma[1], Raman Chauhan[2], Sudhanshu Aggarwal[3]
[1,2,3] *Assistant Professor, Department of Mathematics,*
Noida Institute of Engineering & Technology, Greater Noida, U.P., India
6nidhisharma@gmail.com, raman28chauhan@gmail.com, mathsjunction@gmail.com

***Abstract*--** **This paper considers the waiting of students in colleges at the time of admissions at fee counter as a single-channel queuing system with Poisson arrivals and exponential service rate where service discipline is first come first serve.**

Queue is a common phenomenon which has seen usually in colleges at the time of fee submission. Hence queuing theory which is the mathematical study of waiting lines or queue is suitable to be applied in the fee counter because it is associated with queue and waiting line where students who cannot be served immediately have to queue for service

Keywords: **Queuing process and traffic intensity, Service Pattern,Poisson arrival rate, Exponential service rate, Single channel queuing system, Little's theorem,(M/M/1) Queuing model.**

I. INTRODUCTION

Etymology of Queuing System: The word queue comes, via French, from the Latin Cauda, meaning tail. The spelling "queuing" over "queuing" is encountered in the academic research field.Now-a-days'Queue' is a common word that means a waiting line or the act of joining a line.This paper will take a brief look into the formulation of queuing theory along with model and applications of its use. The goal of the paper is to provide the reader with enough background in order to properly model a basic queuing system.Queuing theory is generally considered a branch of operations research because the results are often used when making business decisions about the resources needed to provide a service.Queuing theory was initially proposed by A.K. Erlang in 1903.The first paper on Queuing theory "The theory of Probabilities and Telephone Conversation" was published in 1909 by A.K. Erlang, Now considered the father of the field.

In everyday life, it is seen that a number of people arrive at a cinema ticket window,in a restaurant especially during lunch and dinner time, in banks etc.Waiting lines are not only the lines of human beings but also the aeroplanes seeking to land at busy airport, ships to be unloaded, cars waiting for traffic lights to turn green etc.Researchers have used queuing theory to model the restaurant operation [3], reduce cycle time in fast food restaurants[4], to increase efficiency in this field also[6].Some problems in the theory of queues were given by Kendal [10]. Klienrock [11-12] has done extensive studies on queue and its applications. Onyesolu and Edebeatu [13] gave a comparative analysis of single server queuing models. Application of queuing theory to patient satisfaction at a tertiary hospital in Nigeria was given by Ameh et.al. [14]. Aradhye and Kullurkar [15] gave the application of queuing theory to reduce waiting period of Pilgrim. Onyesolu and Asogwa [16] studied single server queuing models and birth-death process.

This paper uses queuing theory to study the Service rate and Utilization factor of fee counter of a college.In colleges, students arrive randomly and the service time is also random. We are using M/M/1 queuing model to derive the service rate and Utilization factor of the queue. On average, students are served on weekdays (Monday to Saturday).

II. QUEUING THEORY

Queuing theory is concerned with the statistical description of the behavior of queues with finding the probability distribution of the number in the queue from which the mean and variance of queue length and the probability distribution of waiting time for a customer, or the distribution of a server's busy periods.

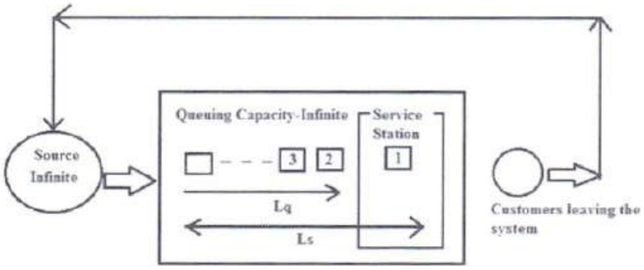
In 1908, Copenhagen Telephone Company requested A. K.Erlang to work on the holding times in a telephone switch. He identified that the number of telephone conversations and telephone holding time fit into Poisson distribution and exponentially distributed. This was the beginning of the study of queuing theory.

III. QUEUING SYSTEM

A queuing system can be completely described by
i) the input or arrival pattern (customers)
ii) the service mechanism (service pattern)
iii) the queue discipline
iv) customer's behaviour

The diagrammatic representation of the queuing system is shown in the figure 1:

**Author for correspondence*



IV. FEE COUNTER MODEL (M/M/I Queuing Model)

Terminology and notation:

λ: The mean students arrival rate

μ: The mean service rate

ρ = Utilization factor or traffic intensity

Probability of zero student in the queue

$P_0 = 1 - \rho$

P_n: The probability of having n studentsin the college :

$P_n = P_0 \rho^n = (1-\rho)\rho^n$

N: number of students in the system

L_S: Expected number of students inthe system

$$L_S = \frac{\rho}{1-\rho}$$

L_q: Expected number of students in the queue

$$L_q = \frac{\rho^2}{1-\rho}$$

W_S: Waiting time of students in the system

$$W_S = \frac{1}{\mu - \lambda}$$

W_q: Waiting time of students in the queue

$$W_q = \frac{\rho}{\mu - \lambda}$$

L: The average number of students in the queue

$$L = \frac{\lambda}{\mu - \lambda}$$

V. OBSERVATION AND DISCUSSION

We have collected the one month daily student data by observation during fee submission time, as shown in Table 1:

Days↓/Week→	I	II	III	IV	Total
Monday	175	168	126	92	561
Tuesday	152	145	120	90	507
Wednesday	141	128	110	79	458
Thursday	122	112	97	71	402
Friday	136	115	97	82	430
Saturday	149	134	121	86	490

From the above figure 1,we can say that, the number of students on Monday and Tuesday come more than other days during the months of fee submission. The busiest period for the fee counter is on Mondays and Tuesdays (10:00 am to 5:00 pm).

VI. CALCULATIONS

We have observed that, during Monday and Tuesday, there are, on average 75 people coming to the college for their fee submission in one hour time period. For this we can derive the arrival rate as:

$$\lambda = \frac{75}{60} = 1.25\ student/minute(spm)$$

We also found out from observation that each student spends 5 minutes on average in the fee counter window; the queue length is around 10 students (L_q) on average.

Theoretically, the average waiting time is $W_q = \frac{L_q}{\lambda} = 8\ minute$

From the calculation, we can see that, the observed actual waiting time does not differ by much when it is compared with the theoretical waiting time.

Now the average number of students in the line,

$$L = 1.25\ spm \times 5\ minutes = 6.25\ students$$

$$\mu = \frac{\lambda(1+L)}{L} = 1.45\ spm$$

So $\rho = \frac{\lambda}{\mu} = 0.862$

This is the probability that, the server, in this case fee counter, is busy to serve the students, during college timing. So, during college timing, the probability of zero students in the counter is

$$P_0 = 1 - \rho = 1 - 0.862 = 0.138$$

counter. By developing a simulation model, we will be able to confirm the results of the analytical model that we develope in this paper. By this paper, we discuss the simulation of fee counter model by taking the data of 80 students.

Table 1

S.No.	Inter arrival Time	Service Time	Arrival Time	Departing Time	W_q	W_s
1	1.22	0.42	0	0.42	0	0.42
2	0.68	0.55	1.22	1.77	0	0.55
3	2.65	0.62	1.9	2.52	0	0.62
4	0.65	0.72	4.55	5.27	0	0.72
5	0.88	0.76	5.2	6.03	0.07	0.83
6	1.02	0.62	6.08	6.7	0	0.62
7	0.44	0.74	7.1	7.84	0	0.74
8	0.86	0.75	7.54	8.29	0	0.75
9	1.22	0.76	8.4	9.27	0.11	0.87
10	3.25	0.66	9.62	10.63	0.35	1.01
11	1.12	0.6	12.87	13.47	0	0.6
12	2.54	0.55	13.99	14.54	0	0.55
13	1.85	0.67	16.53	17.2	0	0.67
14	0	0.71	18.38	19.09	0	0.71
15	0.16	0.76	18.38	19.85	0.71	1.47
16	0.26	0.69	18.54	20.54	1.31	2
17	1.86	0.76	18.8	21.3	1.74	2.5
18	0.22	0.69	20.66	21.99	0.64	1.33

S.No.	Inter arrival Time	Service Time	Arrival Time	Departing Time	W_q	W_s
19	3.25	0.66	20.88	22.65	1.11	1.77
20	1.02	0.7	24.13	24.83	0	0.7
21	0.11	0.68	25.15	25.83	0	0.68
22	1.25	0.71	25.26	26.54	0.57	1.28
23	0.44	0.57	26.51	27.11	0.03	0.6
24	0.86	0.78	26.95	27.89	0.16	0.94
25	0.22	0.74	27.81	28.63	0.08	0.82
26	1.98	0.57	28.03	29.2	0.6	1.17
27	0.48	0.75	30.01	30.76	0	0.75
28	1.13	0.55	30.49	31.31	0.27	0.82
29	4.21	0.74	31.62	32.36	0	0.74
30	2.16	0.72	35.83	36.55	0	0.72
31	0.4	0.63	37.99	38.62	0	0.63
32	0.93	0.72	38.39	39.34	0.23	0.95
33	1.33	0.57	39.32	39.91	0.02	0.59
34	2.16	0.5	40.65	41.15	0	0.5
35	0.32	0.71	42.81	43.91	0	1.1
36	3.11	0.77	43.13	43.9	0.39	0.77
37	1.02	0.65	46.24	46.89	0	0.65
38	0.52	0.68	47.26	47.94	0	0.68
39	3.33	0.52	47.78	48.46	0.16	0.68
40	1.2	0.8	51.11	51.91	0	0.8
41	1.23	0.57	52.31	52.88	0	0.57
42	0.23	0.66	53.54	54.2	0	0.66
43	0.55	0.78	53.77	54.98	0.43	1.21
44	1.27	0.62	54.32	55.6	0.66	1.28
45	0.05	0.58	55.59	56.18	0.01	0.59
46	1.26	0.74	55.64	56.92	0.54	1.28
47	0.65	0.83	56.9	57.75	0.02	0.85
48	2.12	0.77	57.55	58.52	0.2	0.97
49	0.82	0.75	59.67	60.42	0	0.75
50	1.46	0.54	60.49	61.03	0	0.54
51	1.12	0.74	61.95	62.69	0	0.74
52	0.8	0.77	63.07	63.84	0	0.77
53	0.23	0.55	63.87	64.42	0	0.55
54	1.14	0.62	64.1	65.04	0.32	0.94
55	1.34	0.7	65.24	65.94	0	0.7
56	2.23	0.52	66.58	67.1	0	0.52
57	0.71	0.61	68.81	69.42	0	0.61
58	1.25	0.63	69.52	70.15	0	0.63
59	2.55	0.75	70.77	71.52	0	0.75
60	0.19	0.74	73.32	74.06	0	0.74
61	1.08	0.73	73.51	74.79	0.55	1.28
62	0.63	0.61	74.59	75.4	0.2	0.81
63	0.13	0.55	75.22	75.95	0.18	0.73
64	0.75	0.59	75.35	76.54	0.6	1.19
65	0.16	0.63	76.1	77.17	0.44	1.07
66	0.35	0.65	76.26	77.82	0.91	1.56

S.No.	Inter arrival Time	Service Time	Arrival Time	Departing Time	W_q	W_s
67	0.55	0.89	76.61	78.71	1.21	2.1
68	0.2	0.74	77.16	79.45	1.55	2.29
69	1.35	0.56	77.36	80.01	2.09	2.65
70	1.41	0.57	78.71	80.58	1.3	1.87
71	0.22	0.65	80.12	81.2	0.86	1.08
72	0.56	0.63	80.34	81.83	0.86	1.49
73	0.83	0.68	80.9	82.51	0.93	1.61
74	2.11	0.46	81.73	82.97	0.78	1.24
75	0.86	0.51	83.84	84.35	0	0.51
76	2.65	0.64	84.7	85.34	0	0.64
77	0.44	0.84	87.35	88.19	0	0.84
78	1.32	0.52	87.79	88.71	0.4	0.92
79	0.56	0.53	89.11	89.64	0	0.53
80	0.84	0.62	89.67	90.29	0	0.62

By the simulation model, we can say that, that sum of the service time for 80 students is 52.82 minutes. Hence the service time for one customer is 0.6602 minutes. There fore, the service rate μ=1.51 spm, arrival rateλ = 1.13 spm and utilization factor ρ = 0.748

The actual service rate is μ=1.35 spm, arrival rate is λ=1.25. So we can say that, there is no much difference between the actual service rate, arrival rate and the estimated service rates, arrival rate.

VII. CONCLUSIONS

This research paper has discussed the application of queueing theory in the field of fee submission of college.

From the result we have obtained that the rate at which studentsarrive in the queuing system is 1.13 students per minute and the service rate is 1.51 students per minute. The utilization factor for fee counter is very high in Monday and Tuesday during college timing. In this way, this research can contribute to the betterment of a college at the time of fee submission. The result of this paper work may become the reference to analyze the current college system and improve the next system. This research will give a great impact on all colleges' fee counter system for the betterment service to the students.

REFERENCES

[1] V. Sundarapandian (2009) "Probability, Statistics and Queuing Theory", PHI Learning, ISBN 8120338448.

[2] A. Nasroallah(2004), "Monte Carlo simulation of Markov chain steady-state distribution", Extracta Mathematicae, 19(2), 279-288.

[3] D.M. Brann and B.C. Kulick(2002) "Simulation of restaurant operationsusing the restaurant modeling studio", Proceedings of the 2002 Winter Simulation Conference, IEEE Press, 1448-1453.

[4] S.A. Curin, J.S. Vosko, E.W. Chan, and O. Tsimhoni(2005) "Reducing service time at a busy fast food restaurant on campus", Proceedings of the 2005 Winter Simulation Conference, IEEE Press.

[5] N.H. Shah, R.M. Gor, H. Soni(2010)"Operations Research" 4th Edition, ISBN 978-81- 203-3128-0. Eastern Economy Edition, Phi Learning India.

[6] [6]A.K. Kharwat(1991)"Computer Simulation: An important tool in the fast-food industry", Proceedings of the 1991 Winter Simulation Conference, IEEE Press, 811-815.

[7] S.J. Bose(2002) "Chapter1: An introduction to queuing systems", Kluwer/Plenum Publishers.

[8] M. Laguna and J. Marklund(2013)"Business process modeling, simulation and design", second edition ISBN 13: 9781439885253, CRC press, Taylor and Francis group, Boca Raton.

[9] K.Rust(2008)"Using Little's Law to estimate cycle time and cost", Proceedings of the 2008 Winter Simulation Conference, IEEE Press, doi: 10.1109/ WSC.2008.4736323.

[10] D.G. Kendal, "Some problems in the theory of queues", Journal of the Royal Statistical Society 13(1951), 151-185.

[11] L. Kleinrock(1975), "Queuing systems-volume I: Theory, John Wiley and Sons Inc.

[12] L.Kleinrock (1976), "Queuing systems-volume II: Computer applications, John Wiley and Sons Inc.

[13] M.O. Onyesolu and D. Edebeatu (2010) "A comparative analysis of single server queuing models", The Awka Engineer 2, 6-13.

[14] N. Ameh, B. Sabo and M.O. Oyefabi(2013) "Application of queuing theory to patient satisfaction at a tertiary hospital in Nigeria", Niger. Med. J., 54(1), 64-67.

[15] A. Aradhye and S. Kullurkar(2014) "Application of queuing theory to reduce waiting period of Pilgrim", Int. J. of Innovative Research in Science, Engg. &Tech., 2(10), 16775-16781.

[16] M.O. Onyesolu and D.C. Asogwa (2011) "Single server queuing models and Birth-Death process", Comp. Tech. and Appl. 2, 644-649.

Nidhi Sharma received her M.Sc. degree from Banasthali University Rajasthan in 2015. She is working as an Assistant Professor in Noida Institute of Engineering & Technology, Greater Noida. She has also qualified IIT Jam exam (2013).She has published many research papers in national and international journals. Her fields of interest include Optimization, Fuzzy Logic and Topological Spaces.

Raman Chauhan completed MSc. (Mathematics) from Indian Institute of Technology, Roorkee in 2011. He is working at NIET, Greater Noida for the last three years as a dynamic, successful and responsible faculty member in the department of Mathematics. He has also qualified IIT Jam exam (2009) and Gate exam (2011, 2013, 2014 & 2015).

Sudhanshu Aggarwal received his M.Sc. degree from M.S. College, Saharanpur in 2007. He has also qualified CSIR NET examination (June-2010, June-2012, June-2013, June-2014 &June-2015) in Mathematical Sciences. He is working as an Assistant Professor in Noida Institute of Engineering & Technology, Greater Noida. He is equipped with an extraordinary caliber and appreciable academic potency. He has around nine years of teaching experience at various engineering colleges affiliated to AKTU. His fields of interest include Integral Transform Methods, Differential and Partial Differential Equations, Integral Equations and Number Theory.